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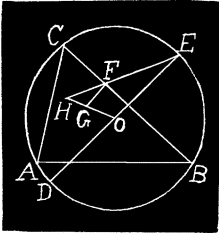
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I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Arkansas-Texas.

Let  $O$  and  $H$  be the circum and ortho-centers respectively of the triangle  $ABC$ . Draw the diameter  $DE$ , connect  $E$  and  $H$ , and from  $F$  the mid-point of  $EH$  draw  $FG$  parallel to  $OE$ .

Now  $H$  and  $O$  are inverse points.  
 $G$  is the mid-point of  $HO$  and  $GF = \frac{1}{2}OE = \text{a constant}$ .  
 $\therefore G$  is the center and  $GF$  the radius of the nine-point circle.  
 $\therefore$  The locus of  $F$  is the nine-point circle.



II. Solution by the PROPOSER.

Let  $l\alpha + m\beta + n\gamma = 0 \dots \dots \dots (1)$  be any diameter. The isogonal transformation of (1) is

$$\frac{l}{\alpha} + \frac{m}{\beta} + \frac{n}{\gamma} = 0 \dots \dots \dots (2).$$

Now (1), passing through the center of the circumcircle, the coordinates of which are proportional to  $\cos A, \cos B, \cos C$ , gives the relation

$$l\cos A + m\cos B + n\cos C = 0 \dots \dots \dots (3).$$

Also, the center of (2), which is an equilateral hyperbola, with condition (3), is given by

$$\frac{l}{n} = \frac{-a\alpha^2 + b\alpha\beta + c\alpha\gamma}{b\beta\gamma - c\gamma^2 + a\alpha\gamma}, \quad \frac{m}{n} = \frac{a\alpha\beta - b\beta^2 + c\beta\gamma}{b\beta\gamma - c\gamma^2 + a\alpha\gamma} \dots \dots \dots (4).$$

Dividing (3) by  $n$ , and substituting equations (4), and reducing,

$$a\beta\gamma + b\alpha\gamma + c\alpha\beta - a\alpha^2\cos A - b\beta^2\cos B - c\gamma^2\cos C = 0 \dots \dots \dots (5),$$

the nine-points circle.

57. Proposed by J. OWEN MAHONEY, B. E., Graduate Fellow and Assistant in Mathematics, Vanderbilt University, Nashville, Tennessee.

Show that pairs of points, on a straight line, may be so related harmonically that a pair of real points will be harmonic with regard to a pair of imaginary points, and by this means prove that there are an indefinite number of conjugate pairs of imaginary points on a real line.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, Ohio.

If the four points be  $A, B, C, D$ , and the axis of  $x$  coincide with the given straight line,  $A, B$  may be supposed given by

$$\alpha x^2 + 2\beta x + \gamma = 0 \dots \dots \dots (1),$$

$$\text{or } x = \frac{-\beta \pm \sqrt{\beta^2 - \gamma^2}}{\alpha} \dots \dots \dots (2),$$

$$\text{and } C, D, \text{ by } \alpha' x^2 + 2\beta' x + \gamma' = 0 \dots \dots \dots (3).$$

Now as long as  $\gamma$  exceeds  $\beta$ , (2) gives imaginary values for  $x$ , and so for a like pair of values for (3), which does not violate the condition

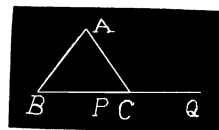
$$\alpha\gamma' + \alpha'\gamma = 2\beta\beta' \dots \dots \dots (4),$$

any number of values of  $\beta, \gamma$  in (2) always being consistent with (4).

II. Solution by JOHN B. FAUGHT, A. M., Instructor in Mathematics, Indiana University, Bloomington, Indiana.

Using trilinear coordinates, take  $B$  and  $C$  for the two real points on the real line  $\alpha=0$ , i. e.,  $b\beta + c\gamma = 2\Delta$ .  $B^2 + K^2\gamma^2 = 0$ , is the equation of two lines through  $A$ ; that is  $\beta + Ki\gamma = 0$ , and  $\beta - Ki\gamma = 0$ . These lines form with  $\beta=0$  ( $AC$ ) and  $\gamma=0$  ( $AB$ ) a harmonic pencil, and hence intersect  $BC$  in two points forming with  $B$  and  $C$  a harmonic range.

Moreover these lines are imaginary for all real values of  $K$  and hence must intersect  $BC$  in imaginary points, otherwise they would contain two real points, which is impossible.



The coordinates of the points of intersections of these imaginary lines may be found by solving with  $b\beta + c\gamma = 2\Delta$ . Thus  $\beta = -Ki\gamma$  gives

$$(c - bKi)\gamma = 2\Delta \text{ and } \gamma = \frac{2\Delta c}{c^2 + b^2 K^2} + \frac{2\Delta Kb}{c^2 + b^2 K^2}i$$

$$\text{and } \beta = \frac{2\Delta K^2 b}{c^2 + b^2 K^2} - \frac{2\Delta Kc}{c^2 + b^2 K^2}i, \text{ and } \beta = Ki\gamma, \text{ gives}$$

$$\gamma = \frac{2\Delta c}{c^2 + b^2 K^2} - \frac{2\Delta Kb}{c^2 + b^2 K^2}i, \text{ and } \beta = \frac{2\Delta K^2 b}{c^2 + b^2 K^2} + \frac{2\Delta Kc}{c^2 + b^2 K^2}i.$$

If  $P$  and  $Q$  denote the imaginary points of intersection, we see that their coordinates are conjugates. These points are called "conjugués harmoniques" with respect to  $B$  and  $C$ , by M. Chasles.

It is evident that by giving different values to  $K$  an infinite number of such points can be found.

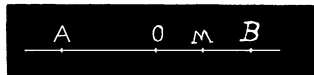
### III. Solution by the PROPOSER.

The roots of  $ax^2 + 2bx + c = 0$  and  $a'x^2 + 2b'x + c' = 0$  will be harmonic if  $ac' + a'c - 2bb' = 0$  (see Scott's Geometry, page 45).

Let  $x^2 = p^2$  give the points  $A$  and  $B$ . Let  $x = OM = K < (OB = p)$  be midway between the other points,  $P$  and  $Q$ . The equation giving  $P$  and  $Q$  is

$$a'x^2 + 2b'x + c' = 0, \text{ with the conditions } \frac{b'}{a} = -K, \text{ and } c' - p^2 a' = 0,$$

$$\text{or } x^2 - 2Kx + p^2 = 0.$$



But since  $K < p$ ,  $K^2 - p^2 < 0$ , the roots of this equation are imaginary, and since there are an indefinite number of values for  $K < p$ , there will be an indefinite number of pairs of imaginary points on the line harmonic with the given real pair. (Scott's Geometry, page 45.)

Solved in a similar manner by *G. B. M. ZERR*.

## PROBLEMS.

63. Proposed by *ALFRED HUME*, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O. University, Mississippi.

Prove, analytically :—A rectangular hyperbola cannot be cut from a right circular cone unless the angle at its vertex is greater than a right angle.

64. Proposed by *WILLIAM E. HEAL*, Member of the London Mathematical Society and Treasurer of Grant County, Marion, Indiana.

Let the bisectors of the angles  $A, B, C$  of a triangle meet the sides opposite  $A, B, C$  in  $A', B', C'$ . Let  $AA', BB', CC'$  meet the sides of the triangle  $A'B'C'$  in  $A'', B'', C''$ . Let this process continue indefinitely. Express the sides and angles of the triangle  $A^{(m)}B^{(m)}C^{(m)}$  in terms of the sides and angles of the original triangle  $ABC$ .